HW 7.4

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Name:_____ **Date:**_____

Math 10 Honours: HW 7.4 Conditional Probability with Tree Diagrams

1. A dice is rolled twice and the sum is taken. What is the probability that the sum is 8?

$(2,6)(3,5)(4,4)(5,3)(6,2) \longrightarrow 5/8$

2. A dice is rolled twice and the sum is taken. If the first roll is a 3, then what is the probability that the sum is 8?

[3] [5]

- IF THE FIRST ROLL IS A 3, THE 2ND ROLL CAN BE 1 TO 6.
- $P(\text{sum}=8 \mid 3) = \frac{1}{6}$

3. Two cards drawn without replacement from a deck of 52 cards. Determine $P(A \text{ and } B)$, if event "A" is the first card is hearts and event "B" is second card is an even number

- THERE ARE 13 HEARTS
- THERE ARE 20 EVEN CARDS 2, 4, 6, 8, 10
- SEVEN, 8 NOT EVEN

$P(A \text{ and } B) = \frac{5}{13}\left(\frac{19}{51}\right) + \frac{8}{13}\left(\frac{20}{51}\right)$

$= \_\_\_\_\_$

4. The following chart shows the number of students getting A's in a class for each gender. A student is chosen randomly from the group. Use the chart to answer the following questions:

	Females	Male
Getting A's	34	24
Not Getting A's	52	65

a) Given that the student is getting A's, what is the probability that the student is a girl?

- 58 STUDENTS GETTING A's
- 34 ARE GIRLS

$P(x) = \frac{34}{58} = \frac{17}{29}$

b) Given that the student is not getting A's, what is the probability that the student is a boy?

- 117 STUDENTS NOT GETTING A's
- 65 ARE BOYS

$P(x) = \frac{65}{117}$

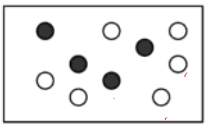
58
117
175

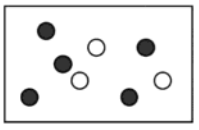
c) What is the probability that the student is getting A's

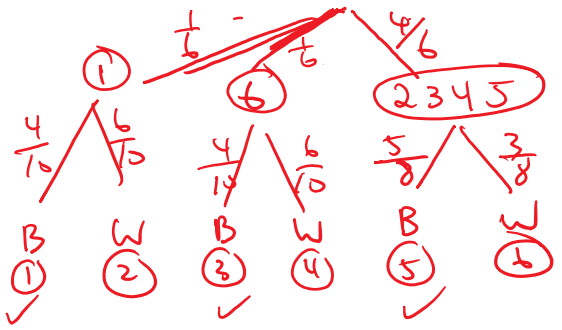
- 175 STUDENTS, 58 GETTING A's

$P(x) = \frac{58}{175}$

5. A single die is rolled. If a 1 or 6 is rolled, a ball is removed from box 1. If a 2, 3, 4 or 5 is rolled, a ball is removed from box 2.


Box 1


Box 2



a) Determine the probability that the ball is white.

$\frac{1}{6}\left(\frac{6}{10}\right) + \frac{1}{6}\left(\frac{6}{10}\right) + \frac{4}{6}\left(\frac{3}{8}\right)$

b) Given that the ball is white, determine the probability that the ball came from box 1.

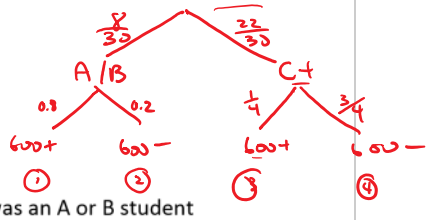
$$\frac{\frac{1}{6}\left(\frac{6}{10}\right)}{\frac{1}{6}\left(\frac{6}{10}\right) + \frac{1}{6}\left(\frac{6}{10}\right) + \frac{4}{6}\left(\frac{3}{8}\right)}$$

c) Given that the ball is black, what is the probability that a "1" was rolled?

$$P(1 \mid \text{BLACK}) = \frac{1}{1+3+3}$$

$$= \frac{\frac{1}{6}\left(\frac{4}{10}\right)}{\frac{1}{6}\left(\frac{4}{10}\right) + \frac{4}{6}\left(\frac{4}{10}\right) + \left(\frac{4}{6}\right)\left(\frac{5}{8}\right)}$$

6. In a class of thirty students, eight are A or B students and the rest are C+ or less students. A or B students score over 600 on SAT math tests 80% of the time. C+ or less students score under 600 on the same test 75% of the time.



a. Determine the probability of a student in the class scoring under 600.

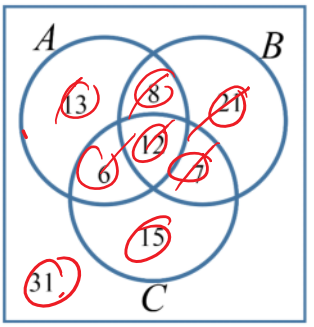
$P(x<600) = ② + ④$

$= \frac{8}{30}\left(\frac{1}{5}\right) + \frac{22}{30}\left(\frac{3}{4}\right)$

b. Given that a student scored under 600, determine the probability that it was an A or B student

$P(x) = \frac{②}{② + ④}$

7. The Venn diagram on the right indicates the number of students taking each subject course "A" Arts, "B" Biology, and "C" Calculus. Use the diagram to find the number of students in each region:



a) $P(B|A) = \frac{20}{39}$

b) $P(C|A)$ $\frac{18}{39}$

C) $P(A|\overline{B})$ $\frac{19}{65}$

d) $P(\overline{B}|A)$ $\frac{19}{39}$

e) $P(\overline{B}|\overline{A}) = \frac{46}{74}$

f) $P(C|\overline{A or C}) = \frac{0}{52} = 0$

g) $P(B|\overline{A \text{ and } C}) = \frac{36}{95}$

h) $P(\overline{B \text{ and } C}|\overline{A or B}) = \frac{46}{46} = 1$

8. The probability that a car battery will last one year is 0.8 and that it will last two years is 0.5. At the end of the first year, what is the probability that it will last until the end of the second year?

0.2.

$P(2|1) = \frac{0.5}{0.8}$

9. A student takes a multiple choice test. All of the questions have a choice of A, B, C or D. There is only one correct answer for each question. The student guessed on two of the questions. Given that at least one of the guesses is correct, what is the probability that both guesses are correct?

Q1	C	W	C
Q2	W	C	C

← ONLY 3 POSSIBLE OUTCOMES

OUT OF 3, ONLY ONE CASE BOTH ARE CORRECT

$P(x) = \frac{1}{3}$.

10. Brad randomly picks 3 distinct numbers from the set {1,2,3,4,5,6,7,8,9} and arranges them in descending order to form a 3 digit number, Steve randomly picks 3 distinct numbers from the set {1,2,3,4,5,6,7,8} and also arranges them in descending order to form a 3 digit number. What is the probability that Brad's number is larger than Steve's? AMC12 2010

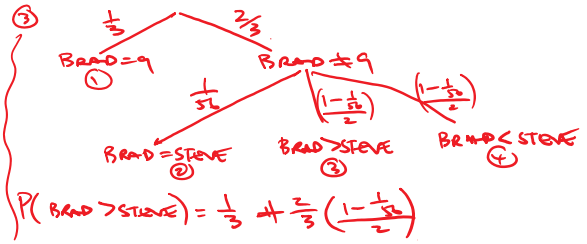
① IF BRAD GETS THE 9, HE WILL BE BIGGER

$P(\text{BRAD: 9}) = \frac{{}_8C_2}{{}_9C_3} = \frac{\frac{8\times7}{1\times2}}{\frac{9\times8\times7}{1\times2\times3}} = \frac{3}{9} = \frac{1}{3}$ ← $\frac{1}{3}$ CHANCE OF SELECTING A 9.

$P(\text{BRAD: } \overline{9}) = 1 - \frac{1}{3} = \frac{2}{3}$ ← $\frac{2}{3}$ OF NOT GETTING A 9.

② BRAD = STEVE. (CHANCES THEY WILL GET SAME #)

$P(\text{EQUAL}) = \frac{1}{{}_8C_3} = \frac{1}{\frac{8\cdot7\cdot6}{1\times2\times3}} = \frac{1}{56}$ ← CHANCE THEY HAVE SAME NUMBER



$P(\text{BRAD} > \text{STEVE}) = \frac{1}{3} + \frac{2}{3}\left(\frac{1-\frac{1}{56}}{2}\right)$

11. For a peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6 on each die are in the ratio of 1:2:3:4:5:6. What is the probability of rolling a total of 7 on the two dice? Amc 2006

X	P(X)
1	1/21
2	2/21
3	3/21
4	4/21
5	5/21
6	6/21

$P(x=7) = \frac{1\times6}{21^2} + \frac{2\times5}{21^2} + \frac{3\times4}{21^2} + \frac{4\times3}{21^2} + \frac{5\times2}{21^2} + \frac{6}{21^2}$

12. A bag contains 40 balls, each of which is black or gold. Fred reaches into the bag and randomly removes two balls. Each ball in the bag is equally likely to be removed. If the probability that two gold balls are removed is 5/12, how many of the 40 balls are gold? Euclid

GOLD: X

BLACK: 40−X

$\frac{X}{40} \times \frac{X-1}{39} = \frac{5}{12}$ → $\frac{X}{10} \times \frac{X-1}{13} = \frac{5}{12}$

$X(X-1) = 10\times13\times5$

$X(X-1) = 26(25)$

$X = 26$

13. Two fake coins of equal weight are mixed with 8 identical real coins. The weight of each of the fake coins is different from the weight of the each of the real coins. A pair of coins is selected at random without replacement from the remaining 8 coins. The combined weight of the first pair is equal to the combined weight of the second pair. What is the probability that all 4 selected coins are real? Amc10

① POSSIBLE CASES: 2.

② CASE #1, BOTH PAIRS ARE REAL.

FF, RRRR, RRFF.

$\frac{{}_8C_2 \times {}_6C_2}{{}_{10}C_2}$ VS. $\frac{{}_8C_4}{{}_{10}C_2}$

$\frac{8\cdot7}{1\times2}\ \frac{6\cdot5}{1\times2}$ $\quad 8\ 7\ 6\ 5$ / $1\ 2\ 3\ 4$

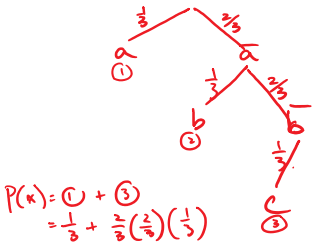
14. Positive integers "a", "b", and "c" are randomly and independently selected with replacement from the set {1,2,3,..... , 2010}. What is the probability that $abc + ab + a$ is divisible by 3? Amc 12 2010

$abc + ab + a = a(bc + b + 1)$

① IF 'a' IS A MULTIPLE OF 3 THEN IT'S ALL DIVISIBLE by 3
$\frac{1}{3}$ OF THE #'s ARE DIVISIBLE BY 3, $\frac{2}{3}$ ARE NOT

② IF 'b' IS A MULTIPLE OF 3, THEN $(bc+b+1)$ CAN NOT DIVIDE by 3

③ IF 'b' IS NOT A MULTIPLE OF 3, THEN 'c' HAS 1/3 CHANCE FOR $(bc+b+1)$ TO BE DIVISIBLE by 3.



$P(K) = ① + ③$
$= \frac{1}{3} + \frac{2}{3}\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$

15. Challenge: Six ants simultaneously stand on the six vertices of a regular octahedron, with each ant at a different vertex. Simultaneously and independently, each ant moves from its vertex to one of the four adjacent vertices, each with equal probability. What is the probability that no two ants arrive at the same vertex? AMC 12 2005

16. Box 1 contains one gold marble and one black marble. Box 2 contains one gold marble and two black marbles. Box 3 contains one gold marble and three black. Whenever a marble is chosen randomly from one of the boxes, each marble in that box is equally likely to be chosen. A marble is randomly chosen from Box 1 and placed in Box 2. Then a marble is randomly chosen from box 2 and placed in Box 3. Finally, a marble is randomly chosen from Box 3. What is the probability that the marble chosen from Box 3 is gold? Fermat